1. (a) Simplify fully

$$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}$$

(3)

Given that

$$\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15), \quad x \neq -5,$$

(b) find x in terms of e.

(4)
(Total 7 marks)

2. (i) Find the exact solutions to the equations

(a) $\ln(3x - 7) = 5$

(3)

(b) $3^x e^{7x+2} = 15$

(5)

(ii) The functions f and g are defined by

$$f(x) = e^{2x} + 3, \qquad x \in \mathbb{R}$$

$$g(x) = \ln(x - 1), \qquad x \in \mathbb{R}, x > 1$$

(a) Find f^{-1} and state its domain.

(4)

(b) Find fg and state its range.

(3)
(Total 15 marks)

3. Find the exact solutions to the equations

(a) $\ln x + \ln 3 = \ln 6$,

(2)

(b) $e^x + 3e^{-x} = 4$.

(4)
(Total 6 marks)

4. Find, giving your answer to 3 significant figures where appropriate, the value of x for which

(a) $3^x = 5$,

(3)

(b) $\log_2 (2x + 1) - \log_2 x = 2$,

(4)

(c) $\ln \sin x = -\ln \sec x$, in the interval $0 < x < 90°$.

(3)
(Total 10 marks)

5. Given that $y = \log_a x$, $x > 0$, where a is a positive constant,

(a) (i) express x in terms of a and y,

(1)

(ii) deduce that $\ln x = y \ln a$.

(1)

(b) Show that $\dfrac{dy}{dx} = \dfrac{1}{x \ln a}$.

(2)

The curve *C* has equation $y = \log_{10} x$, $x > 0$. The point *A* on *C* has *x*-coordinate 10. Using the result in part (b),

(c) find an equation for the tangent to *C* at *A*.

(4)

The tangent to *C* at *A* crosses the *x*-axis at the point *B*.

(d) Find the exact *x*-coordinate of *B*.

(2)
(Total 10 marks)

6. (a) Simplify $\dfrac{x^2 + 4x + 3}{x^2 + x}$.

(2)

(b) Find the value of *x* for which $\log_2 (x^2 + 4x + 3) - \log_2 (x^2 + x) = 4$.

(4)
(Total 6 marks)

1. (a) $\frac{(x+5)(2x-1)}{(x+5)(x-3)} = \frac{(2x-1)}{(x-3)}$ M1 B1 A1 aef 3

Note

M1: An attempt to factorise the numerator.
B1: Correct factorisation of denominator to give $(x + 5)(x - 3)$.
Can be seen anywhere.

(b) $\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$ M1

$\frac{2x^2 + 9x - 5}{x^2 + 2x - 15} = \mathrm{e}$ dM1

$\frac{2x - 1}{x - 3} = \mathrm{e} \Rightarrow \quad 3\mathrm{e} - 1 = x(\mathrm{e} - 2)$ M1

$\Rightarrow x = \frac{3\mathrm{e} - 1}{\mathrm{e} - 2}$ A1 aef cso 4

Note

M1: Uses a correct law of logarithms to combine at least two terms.
This usually is achieved by the subtraction law of logarithms to give

$\ln\left(\frac{2x^2 + 9x - 5}{x^2 + 2x - 15}\right) = 1$

The product law of logarithms can be used to achieve
$\ln(2x^2 + 9x - 5) = \ln(\mathrm{e}(x^2 + 2x - 15))$.
The product and quotient law could also be used to achieve

$\ln\left(\frac{2x^2 + 9x - 5}{e(x^2 + 2x - 15)}\right) = 0$

dM1: Removing ln's correctly by the realisation that the anti-ln of 1 is e.
Note that this mark is dependent on the previous method mark being awarded.
M1: Collect x terms together and factorise.
Note that this is not a dependent method mark.

A1: $\frac{3e-1}{e-2}$ or $\frac{3e^1-1}{e^1-2}$ or $\frac{1-3e}{2-e}$. **aef**

Note that the answer needs to be in terms of e. The decimal answer is 9.9610559…

Note that the solution must be correct in order for you to award this final accuracy mark.

[7]

2. (i) (a) $\ln(3x - 7) = 5$

$\mathrm{e}^{\ln(3x - 7)} = \mathrm{e}^5$ Takes e of both sides of the equation.

		This can be implied by $3x - 7 = e^5$.	M1	
		Then rearranges to make x the subject.	dM1	
	$3x - 7 = e^5 \Rightarrow$			
	$x = \dfrac{e^5 + 7}{3}\{= 51.804...\}$	***Exact answer*** of $\dfrac{e^5 + 7}{3}$.	A1	3
(b)	$3^x e^{7x+2} = 15$			
	$\ln(3^x e^{7x+2}) = \ln 15$	Takes ln (or logs) of both sides of the equation.	M1	
	$\ln 3^x + \ln e^{7x+2} = \ln 15$	Applies the addition law of logarithms.	M1	
	$x \ln 3 + 7x + 2 = \ln 15$	$x \ln 3 + 7x + 2 = \ln 15$	A1 oe	
	$x(\ln 3 + 7) = -2 + \ln 15$	Factorising out at least two x terms on one side and collecting number terms on the other side.	ddM1	
	$x = \dfrac{-2 + \ln 15}{7 + \ln 3}\{= 0.0874...\}$	***Exact answer*** of $\dfrac{-2 + \ln 15}{7 + \ln 3}$	A1 oe	5
(ii) (a)	$f(x) = e^{2x} + 3, x \in$ ℝ			
	$y = e^{2x} + 3 \Rightarrow y - 3 = e^{2x}$	Attempt to make x (or swapped y) the subject	M1	
	$\Rightarrow \ln(y - 3) = 2x$			
	$\Rightarrow \frac{1}{2}\ln y - 3 = x$	Makes e^{2x} the subject and takes ln of both sides	M1	
	Hence $\underline{f^{-1}(x) = \frac{1}{2}\ln(x - 3)}$	$\frac{1}{2}\ln(x-3)$ or $\underline{\ln\sqrt{(x-3)}}$ or $\underline{f^{-1}(x) = \frac{1}{2}\ln(y-3)}$ (see appendix)	<u>A1</u> **cao**	
	$f^{-1}(x)$: Domain: <u>$x > 3$</u> or <u>$(3, \infty)$</u>	Either <u>$x > 3$</u> or <u>$(3, \infty)$</u> or <u>Domain > 3.</u>	B1	4

(b) $g(x) = \ln(x - 1), x \in$ □, $x > 1$

$fg(x) = e^{2\ln(x-1)} + 3$

$\{= (x - 1)^2 + 3\}$ — An attempt to put function g into function f. M1

$e^{2\ln(x-1)} + 3$ or $(x - 1)^2 + 3$ or $x^2 - 2x + 4$. A1 isw

$fg(x)$: Range: $\underline{y > 3}$ or $\underline{(3, \infty)}$ — Either $\underline{y > 3}$ or $\underline{(3, \infty)}$ or $\underline{\text{Range} > 3}$ or $\underline{fg(x) > 3}$. B1 3

[15]

3. (a) $\ln 3x = \ln 6$ or $\ln x = \ln\left(\frac{6}{3}\right)$ or $\ln\left(\frac{3x}{6}\right) = 0$ M1

$x = 2$ (only this answer) A1cso 2

Answer $x = 2$ with no working or no incorrect working seen: M1A1

Note: $x = 2$ from $\ln x = \frac{\ln 6}{\ln 3} = \ln 2$ M0A0

$\ln x = \ln 6 - \ln 3 \Rightarrow x = e^{(\ln 6 - \ln 3)}$ allow M1, $x = 2$ (no wrong working) A1

(b) $(e^x)^2 - 4e^x + 3 = 0$ (any 3 term form) M1

$(e^x - 3)(e^x - 1) = 0$

$e^x = 3$ or $e^x = 1$ Solving quadratic M1dep

$x = \ln 3$, $x = 0$ (or ln 1) M1A1 4

1st M1 for attempting to multiply through by e^x : Allow y, X, even x, for e^x

2nd M1 is for solving quadratic as far as getting two values for e^x or y or X etc

3rd M1 is for converting their answer(s) of the form $e^x = k$ to $x = \ln k$ (must be exact)

A1 is for ln3 **and** ln1 or 0 (Both required and no further solutions)

[6]

4. (a) $\log 3^x = \log 5$ M1

taking logs

$x = \frac{\log 5}{\log 3}$ or $x \log 3 = \log 5$ A1

$= 1.46$ cao A1 3

(b) $2 = \log_2 \frac{2x+1}{x}$ M1

$\frac{2x+1}{x} = 4$ or equivalent; B1

4

$2x + 1 = 4x$ M1

multiplying by x to get a linear equation

$x = \frac{1}{2}$ A1 4

(c) $\sec x = 1/\cos x$ B1

$\sin x = \cos x \Rightarrow \tan x = 1$ $x = 45$ M1, A1 3

use of tan x

[10]

5. (a) (i) $\underline{x = a^y}$ B1 1

(ii) In both sides of (i) i.e $\ln x = \ln a^y$ or (y =) $\log_a x = \frac{\ln x}{\ln a}$

$= \underline{y \ln a}$ * $\Rightarrow y \ln a = \ln x$ $B1_{cso}$ 1

B1 $x = e^{y\ln a}$ is BO

B1 Must see $\ln a^y$ or use of change of base formula.

(b) $y = \frac{1}{\ln a} \bullet \ln x, \Rightarrow \frac{dy}{dx}, = \underline{\frac{1}{\ln a} \times \frac{1}{x}}$ * M1, $A1_{cso}$ 2

ALT. $\left[\text{or} \quad \frac{1}{x} = \frac{dy}{dx} \cdot \ln a \quad , \Rightarrow \quad \underline{\frac{dy}{dx} = \frac{1}{x \ln a}} \quad * \right]$

M1, $A1_{cso}$ M1 needs some correct attempt at differentiating.

(c) $\log_{10} 10 = 1 \Rightarrow$ A is (10, $\underline{1}$) $y_A = 1$ B1

from(b) $\underline{m = \frac{1}{10 \ln a} \text{ or } \frac{1}{10 \ln 10}}$ or 0.043 (or better) B1

equ of target $y - 1 = m(x - 10)$

i.e $\underline{y - 1 = \frac{1}{10 \ln 10}(x - 10) \text{ or } y = \frac{1}{10 \ln 10} x + 1 - \frac{1}{\ln 10}}$ (o.e) A1 4

B1 Allow either

M1 ft their y_A and m

(d) $y = 0$ *in* (c) $\Rightarrow 0 = \dfrac{x}{10\ln 10} + 1 - \dfrac{1}{\ln 10} \Rightarrow x, = 10 \ln 10\left(\dfrac{1}{\ln 10} - 1\right)$ M1

$\underline{x = 10 - 10 \ln 10}$ or $\underline{10(1 - \ln 10)}$ or $\underline{10\ln 10(\dfrac{1}{\ln 10} - 1)}$ A1 2

M1 Attempt to solve correct equation. Allow if a not = 10.

[10]

6. (a) $\dfrac{x^2 + 4x + 3}{x^2 + x} = \dfrac{(x+3)(x+1)}{x(x+1)}$ M1

Attempt to factorise numerator or denominator

$= \underline{\dfrac{x+3}{x}}$ or $1 + \dfrac{3}{x}$ or $(x + 3)x^{-1}$ A1 2

(b) LHS = $\log_2\left(\dfrac{x^2 + 4x + 3}{x^2 + x}\right)$ M1 (*)

Use of log a – log b

RHS = 2^4 or 16 B1

$x + 3 = 16x$ M1 (*)

Linear or quadratic equation in x

(*) *dep*

$x = \underline{\tfrac{3}{15}}$ or $\underline{\tfrac{1}{5}}$ or $\underline{0.2}$ A1 4

[6]

1. This question was well answered with candidates usually scoring either 3 marks (about 21%), or 5 marks (about 17%) or all 7 marks (about 46%).

The vast majority of candidates achieved all three marks in part (a). A significant minority of candidates used an alternative method of long division and were invariably successful in achieving the result of $2 + \dfrac{5}{(x-3)}$.

The laws of logarithms caused problems for weaker candidates in part (b). Common errors including some candidates simplifying $\ln(2x^2 + 9x - 5) - \ln(x^2 + 2x - 15)$ to $\dfrac{\ln(2x^2+9x-5)}{\ln(x^2+2x-15)}$ or other candidates manipulating the equation $\ln(2x^2 + 9x - 5) = 1 + \ln(x^2 + 2x - 15)$ into $2x^2 + 9x - 5 = e^1 + x^2 + 2x - 15$. Perhaps more disheartening was the number of candidates who were unable to make x the subject after correctly achieving $\dfrac{2x-1}{x-3} = e$, with some leaving a final answer of $x = \dfrac{1+ex-3e}{2}$. Those candidates who used long division in part (a) usually coped better with making x the subject in part (b).

2. A large majority of candidates were able to gain all 3 marks in (i)(a). The most common mistake was for candidates to make a slip when rearranging to give an answer of $\frac{1}{2}(e^5 - 7)$. A small minority of candidates, however, incorrectly manipulated $\ln(3x - 7) = 5$ to give either $\ln 3x - \ln 7 = 5$ or $3x - 7 = \ln 5$. A few candidates gave the decimal answer of 51.804..., rather than the exact answer.

A significant number of good candidates struggled with (i)(b), thus making this question discriminating for capable candidates. The initial task of taking ln's was often done incorrectly with some candidates taking ln's but ignoring the 3^x term and other candidates replacing 3^x by 3. A significant number of candidates attempted to combine the indices to obtain the incorrect equation of $3e^{8x+2} = 15$. Another common error was for some candidates to write $\ln 3^x(7x + 2) = \ln 15$. A number of candidates who wrote $7x + 2 = \ln\left(\dfrac{15}{3^x}\right)$, could not proceed any further.

Some candidates often left solutions with x appearing on both sides and a small number who successfully gained the first three marks did not know how to proceed further. Candidates displayed total confusion at how to deal with the logarithm of a product and their general logarithmic manipulation was very poor. Many of them seemed completely lost as to how to approach the question, often attempting it several times. Having said this, examiners are pleased to report that a significant minority of candidates were able to obtain the correct answer with ease. Very few of these candidates were able to use elegant methods to arrive at the correct answer.

Part (ii)(a) was generally very well done with many candidates scoring at least 3 of the 4 marks available. Most candidates recognised the need for a "swapped y" method and were able to obtain a correct inverse function. The order of operations was not as well understood by a significant minority of candidates who usually found an inverse of $\dfrac{\ln x - \ln 3}{2}$.

Although a significant minority of candidates were able to correctly state the domain, of those candidates who attempted to write down a domain, common incorrect responses included $x \geq 3$,

$x \in \mathbb{R}$ or giving the domain in terms of *y*.

A majority of candidates found some success with finding the composite function fg(x) in part (ii)(b) and it was pleasing to see how many candidates understood the order of composition. Many candidates gained first 2 marks for initial form of fg(x) but struggled to simplify this, but they were not penalised by the mark scheme for doing so. A minority of candidates with poor bracketing lost the accuracy mark for fg(x) . The range was as poorly answered with some candidates not appreciating the relevancy of the domain of f (x) in answering this question. There were a number of capable candidates who incorrectly stated the range of fg (x) ≥ 3.

3. There were some good solutions to this question, but generally this was a very poor source of marks; fully correct solutions were seen only from the better candidates In part (a), which was intended as a "nice" starter, statements like $x + 3 = 6$ and $\ln x = \ln 6 - \ln 3 = \frac{\ln 6}{\ln 3}$ were quite common, and even candidates who reached the stage $\ln x = \ln 2$ did not always produce the correct answer $x = 2$; $x = e^2$ and $x = 1.99..$, from $x = e^{0.693}$, were not uncommon.
However, it was part (b) where so much poor work was seen; the fact that this required to be set up as a quadratic in e*x* was missed by the vast majority of candidates. Besides the serious error referred to in the introduction the following is a small selection of the more common "solutions" seen: $e^x + 3e^{-x} = 4 \Rightarrow e^x(1 + 3^{-1}) = 4$ (which fortuitously gave $x = \ln 3$);
$e^x + 3e^{-x} = 4 \Rightarrow e^x(1 + 3e^{-2x}) = 4 \Rightarrow e^x = 4$ or $e^{-2x} = 1$;
$e^x + 3e^{-x} = 4 \Rightarrow e^{2x} - 4e^x = -3 \Rightarrow e^x(e^x - 4) = -3 \Rightarrow x = \ln\left(\frac{-3}{e^x - 4}\right)$.

4. The majority of candidates gained the marks in part (a) although a few did not giver their answer to 3 significant figures. Part (b) was well answered by those who understood logs. Most did combine the logs correctly, but some did still split it up into $\log_2 2x + \log_2 x$ or $\frac{\log_2(2x+1)}{\log_2 x}$. Many candidates found the combination of logs and trig functions beyond them.
$\sin x = -1/\sec x$ was a frequent indicator of poor understanding, though many did display they knew $\sec x = 1/\cos x$. Quotient lines often slipped, ln 1/cos x becoming 1/ln cos *x*.

5. Part (a) (i) was usually answered well but the provision of the answers in the next two parts meant that a number of candidates failed to score full marks either through failing to show sufficient working, or by the inclusion of an incorrect step or statement.

The most successful approach to part (b) started from $y = \frac{\ln x}{\ln a}$ but there was sometimes poor use of logs such as $\frac{\ln x}{\ln a} = \ln x - \ln a = \ln\left(\frac{x}{a}\right)$; incorrect notation such as $\frac{dy}{dx}\ln x = \frac{1}{x}$,or errors in differentiation when $\frac{1}{a}$ appeared.

In part (c) the process for finding an equation for the tangent was usually well known but some candidates did not appreciate that the gradient should be a constant and gave a non-linear equation for their tangent. Some confused ln10 with $\log_{10}10$. Many candidates had problems working exactly in the final part and others ignored this instruction and gave an answer of –13.02.

6. Part (a) was a straightforward start to the paper with most candidates able to factorise and simplify perfectly. A few attempted to use long division but this often led to errors, and some weaker candidates simply cancelled the x^2 in the numerator and denominator of the expression.

The second part of this question turned out to be more testing, and it was clear that a number of candidates were not fully familiar with the rules of logarithms. Simply crossing out $\log_2$ was sometimes seen and a large number of candidates thought that $\log a - \log b \equiv \frac{\log a}{\log b}$. Those successfully reaching $\log_2\left(\frac{x+3}{x}\right)$ then had difficulty relating the base 2 with the 4 and common errors included 4^2 $\log_2 4$, 4 log e. Even those who successfully arrived at $15x = 3$ were not yet home and dry, as a surprising number concluded that $x = 5$.